Study of the nuclear medium by $^{12}C + ^{12}C$ elastic scattering analysis at low energy region

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ABSTRACT

The nuclear medium is investigated by studying the $^{12}C + ^{12}C$ elastic scattering at the low energies in the framework of optical model (OM) potential. Both frozen and adiabatic density approximations are used for the description of the nuclear medium during the colliding process. In the OM calculation, the double folding procedure using the realistic CDM3Y3 effective nucleon-nucleon (NN) interactions and the wave functions of colliding nuclei is employed to describe the nucleus-nucleus potential at low energy region below 10 MeV per nucleon. The obtained results from the elastic scattering analyses show that the adiabatic density approximation is more reasonable than the frozen density approximation to describe the overlapping density (the so-called nuclear medium) for the $^{12}C + ^{12}C$ system at low energy region.

Key words: *elastic scattering, Optical Model, adiabatic approximation*

INTRODUCTION

The $^{12}C + ^{12}C$ reaction is an interesting topic that has attracted many researches in both the experimental and theoretical fields over four decades. There are many studies in the experimental field to measure the data over a wide range of energies [1–7], which are analysed theoretically by employing both the phenomenological and microscopic potentials [8–11]. The experimental data of angular distributions corresponding to the region of energies above 10 MeV per nucleon have been studied and explained unambiguously using the optical potential with the deep real part [8]. However, in the energy region below 10 MeV per nucleon, the experimental data have not been analysed clearly, especially at the backward angles and the solutions of the mean field encountered during the traversing of the projectile and the target still remain ambiguities. In addition, the $^{12}C + ^{12}C$ fusion process at energies near and below the Coulomb barrier plays an important role in studying the Carbon-burning process. The reaction is known as a main chain to yield heavier elements in stars and directly relates to the evolution of universe, which has attracted a large number of interest in the nuclear astrophysics up to now [6–7, 11–16]. Unfortunately, the presence of resonant peaks in the $^{12}C + ^{12}C$ reaction cross section challenges both the theoretical and experimental efforts to extrapolate the precise reaction cross sections down to the sub-Coulomb energies and the origin of these resonances still has not been understood obviously. From the brief survey, one can see the mean field behind the $^{12}C + ^{12}C$ reaction at low energy region is still a question that waits for the

ability of realistic theoretical models to investigate and explain clearly.

In principle, the nuclear mean field formed during the di-nuclear collision can be studied by the microscopic potential. In fact, the folding model, which is obtained by averaging an appropriate nucleon-nucleon (NN) interaction over the matter distributions [8, 17–20], is an appropriate approach for this purpose. Many analyses focus on the study of the effective interaction and nuclear distribution, which are known as the important inputs of the folding model [8, 17–20]. Besides, the nuclear medium, which is defined as the nuclear environment around the interaction between two nucleons, is an important physical ingredient of the mean field needed to investigate with the aspect for understanding details the real regime of the $^{12}C + {}^{12}C$ reaction process.

In this paper, based on the optical model (OM) analysis, two kinds of frozen and adiabatic approximations are employed in the framework of double folding potential to investigate the nuclear medium during the collision process of two interacting nuclei at low energies. The microscopic nuclear potentials used for this purpose is constructed with the two-parameter Fermi distributions of nuclear densities [21] and the new version of density dependent (or density dependence in a consistent way; CD) NN interaction [18] that was based on the M3Y (Michigan Three Yukawa) interaction developed by Michigan State University group, so called the CDM3Yn (n=1..6) interaction [18].

In the next section, we discuss the theory of OM potential and double folding model (DFM). Some obtained results of the nucleus-nucleus potentials, the angular distribution analyses and the medium investigation are given in the section of results and discussions. We summarize and conclude in the last section.

METHOD

Optical model

In general, the quantum scattering of incident particles from a target is described by the differential cross section [22]

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 . \quad (1)$$

The value of scattering amplitude $f(\theta)$ is determined according to the OM by solving the Schrodinger equation for the elastic nucleus - nucleus scattering

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + U(r)\right]\psi = E\psi , \quad (2)$$

in here, $\mu = \frac{m_A m_a}{m_A + m_a}$ is the reduced mass (A, a are the labels of the projectile and target nuclei, respectively), U(r) is the complex potential given in the form

$$U(r,E) = V_R(r,E) + iW_I(r,E) + V_C(r). \quad (3)$$

V_C is the Coulomb potential. The nuclear potential includes $V_R(r,E)$ and $W_I(r,E)$ which correspond to the real and imaginary components. We calculate $V_R(r,E)$ by applying the folding model and $W_I(r,E)$ part with using the phenomenological Woods-Saxon shape given in form

$$W_I(r,E) = \frac{W_0(E)}{1+\exp(\frac{r-R_I}{a_I})} . \quad (4)$$

Double folding potential

The nucleus-nucleus interaction, known as the important input for OM calculation, is constructed from the nucleon degrees of freedom in the framework of DFM. From this point of view, the nuclear potential V_R is assumed as the sum of the effective NN interactions v_{ij} and given by the formula below

$$V_R = \sum_{i\in a, J\in A} \langle ij|v_{ij}|ij\rangle = \sum_{i\in a, J\in A} \left\{\langle ij|v_D|ij\rangle + \langle ij|v_{EX}|ji\rangle\right\} = V_{aA}^{D} + V_{aA}^{EX} , \quad (5)$$

with V_{aA}^{D} and V_{aA}^{EX} are the direct and exchange terms, respectively

$$V_{aA}^{D}(\vec{R},E)=\int \rho_a(\vec{r}_a)\rho_A(\vec{r}_A)v_D(\rho,E,s)d^3r_a d^3r_A, \quad (6)$$

$$V_{aA}^{EX}(\vec{R},E)=\int \rho_a(\vec{r}_a,\vec{r}_a+\vec{s})\rho_A(\vec{r}_A,\vec{r}_A-\vec{s})v_{EX}(\rho,E,s)\exp\left[\frac{iK(\vec{R})s}{\mu}\right]d^3r_a d^3r_A. \quad (7)$$

Here $s=\left|\vec{r}_A-\vec{r}_a+\vec{R}\right|$ is the relative distance between two interacting nucleons. r_A, r_a are the nucleon coordinates inside the body target A and projectile a, correspondingly. R refers to the separation of two nuclear centers while E is the energy in the center of mass system. K is defined as the relative momentum. v_D and v_{EX} are the direct and exchange terms of the effective NN interaction, respectively.

Two important inputs for the calculation of the DFM potential are the nuclear densities and the effective NN interactions. The appropriate two-parameter Fermi distributions are used in this work for describing the ground-state nuclear densities of the projectile and target

$$\rho_{a(A)}=\rho_{0a(A)}\left\{1+\exp\left(\frac{r-c_{a(A)}}{d_{a(A)}}\right)\right\}^{-1}, \quad (8)$$

with the set of parameters ($\rho_{0a(A)}=0.194\,fm^{-3}, c_{a(A)}=2.214\,fm, d_{a(A)}=0.425\,fm$) chosen to reproduce correctly the empirical nuclear root-mean-square (r.m.s) radius which is extracted from the elastic electron scattering [21]. While the version of CDM3Y3 giving the best description to ^{12}C + ^{12}C elastic scattering data at intermediate energies [19] has been defined as

$$v_{D(EX)}=g(k)F(\rho).v_{D(EX)}(s), \quad (9)$$

where the radial dependences of CDM3Y3 interactions are defined as a sum of three Yukawa functions with parameters adjusted to reproduce the G-matrix elements in an oscillator basis [20, 23]

$$v_D(s)=11061.625\frac{e^{-4s}}{4s}-2537.5\frac{e^{-2.5s}}{2.5s}, \quad (10)$$

$$v_{EX}(s)=-1524.25\frac{e^{-4s}}{4s}-518.75\frac{e^{-2.5s}}{2.5s}-7.8474\frac{e^{-0.7072s}}{0.7072s} \quad (11)$$

Besides, $F(\rho)$ is known as the function which emphasizes that the NN interaction between two nucleons inside the target and projectile respectively is not equivalent to that in the free space. It should be strongly taken into account the effect of surrounding nucleons, the so-called the medium effect. Consequently, $F(\rho)$ is given in the form

$$F(\rho)=C[1+\alpha\exp(-\beta\rho)+\gamma\rho]+C'[\alpha'\exp(-\beta'\rho)-1]. \quad (12)$$

These parameters ($C=0.2985$, $\alpha=3.4528$, $\beta=2.6388\,fm^{-3}$, $\gamma=-1.5\,fm^{-3}$, $C'=0.38$, $\alpha'=1$, $\beta'=4.484\,fm^{-3}$) are chosen to yield a nuclear incompressibility K value of 217 MeV while still reproduce correctly the saturation properties of nuclear matter such as the saturation density and binding energy [18]. In particular, these parameters $(C',\ \alpha',\ \beta')$ are added to the earlier CDM3Y3 verssion [17, 18] to take into account the rearrangement effect that arises naturally from Landau's theory for many Fermion system [19].

Treatment for nuclear medium

Frozen density approximation

In the present double folding calculations, there are several descriptions for the overlapping densities, which are known as the nuclear environment around the NN interaction between two nucleons, the so-called nuclear medium. The widely used approximation of the nuclear medium is defined as the sum of the local densities of projectile and target corresponding to

the individual positions of the interacting nucleons

$$\rho = \rho_a(\vec{r_a}) + \rho_A(\vec{r_A}). \quad (13)$$

This approximation, which is called the "frozen density approximation" (FDA), is successful for describing the overlapping density in many cases of di-nuclei interactions at the intermediate and high energy regions [8, 17-19].

Adiabatic density approximation

In the low energy region, which the approaching speed of the target and projectile nuclei is slower in comparison with the nucleons' speed in these body nuclei, it is enough time for nucleons inside the compound system to rearrange their single energy levels to make the total system energy as the lowest as possible during the penetration. In this circumstance, the overlapping density or nuclear medium is assumed to change gradually and not exceed the nuclear compound density at the central point that is described realistically by the adiabatic density approximation (ADA). In this regime, the parameters of the Fermi distributions in Eq. (8) are changed instead of being constants in the case of FDA as follows [24]

$$C_{dau}(R) = \begin{cases} C_{dau}(\infty)\exp\left\{\ln\left(\frac{C_{par}}{C_{dau}(\infty)}\right)\left(\frac{R}{R_{cut}}\right)^2\right\} & R \le R_{cut} \\ C_{dau}(\infty) & R > R_{cut} \end{cases}. \quad (14)$$

The sub-label "dau" (or "par") represents for the ^{12}C daughter (or the ^{24}Mg) nuclei and R_{cut} is the distance where two nuclear centers are almost separated. The formula is similar to the parameter *a*. The normalization condition is used for calculating the saturation density ρ_0 in Eq. (8) as follows

$$4\pi\int_0^\infty \rho_{a(A)}(r)r^2dr = a(A). \quad (15)$$

RESULTS AND DISCUSSION

In Fig. 1, we present the results of the overlapping densities with both frozen and adiabatic density approximations which are calculated by the formulae in Eqs. (13)–(15). In the case of ADA, R_{cut} is chosen at which the reorganization of the central part of the compound nucleus into two central densities of the individual daughters starts to occur. In this situation, the separation between two daughter nuclei, R_{cut}, is roundly equal to 3.8 fm [24]. One can see in Fig. 1, the FDA overlapping density at the central point reaches to twice the saturation density of the ^{12}C individual daughter nucleus (~0.388 fm^{-3}) while that of the ADA calculations is equivalent to the ^{24}Mg compound nucleus density (~ 0.167 fm^{-3}). The results point out that the overlapping density in the adiabatic regime changes gradually during the collision process and its component densities tart to dilute at the contact point in order to merge easily two daughter nuclei into each other. In contrast, the FDA overlapping density alters quickly with a tendency to make the compound nucleus more tightly, which causes two daughter nuclei difficult to penetrate each other at the short distance in low energy region.

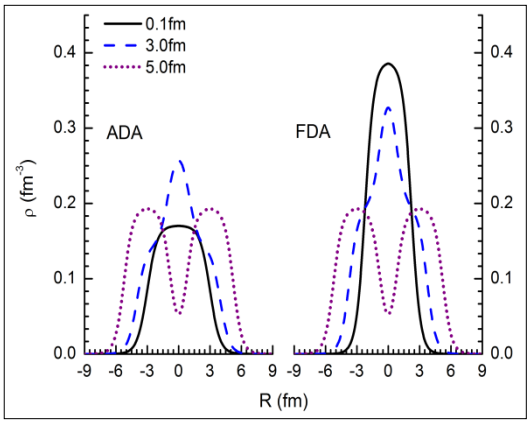

Fig. 1. The overlapping densities within both frozen and adiabatic density approximations as the functions of the relative distance between two daughter nuclei

Within the frozen and adiabatic density regimes, the real parts of nuclear optical potentials are constructed in the framework of the double folding model that are called the FDA and ADA potentials, respectively. Two inputs for the double folding calculation are the nuclear densities of interacting nuclei and the effective NN interactions. In recent work, the realistic two-parameter Fermi distributions and the extended (new) version of the energy, density dependent CDM3Y3 interaction are used to calculate the nuclear potential through Eqs. (5)–(7). As illustrated in Fig. 2, the dashed line presents for the FDA potential and the remaining is shown by the solid line. The results show that the ADA potential drops sharply and is deeper than the FDA potential around 80 MeV at the bombarding energy of 78.8 MeV. We note that from the contact point of R_{cut} ~ 3.8 fm inward, the calculated potentials depend strongly on the choice of ADA and FDA while both of approximations produce the same potentials outside this point. This means that the ADA and FDA generate different medium effects at the short interacting distance that impact critically on the potential strength, known as a distinct feature of each interaction system. It is now to be seen whether these Hartree-Fock type potentials can describe the angular distributions of the $^{12}C + ^{12}C$ system at energies below 10 MeV per nucleon.

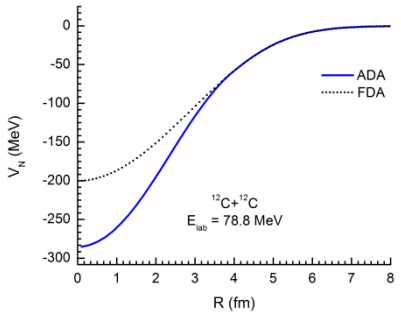

Fig. 2. The real parts of optical potentials for the $^{12}C+^{12}C$ system at the bombarding energy of 78.8 MeV with two frozen and adiabatic density approximations

To investigate the nuclear medium during the collision process of the $^{12}C + ^{12}C$ system at low energies, the optical model calculations are employed to yield the elastic angular distributions. In this model, the microscopic real potentials corresponding to the frozen and adiabatic density approximations are calculated by using the double folding model while the imaginary parts are described by Woods-Saxon shape with parameters adjusted to best fit the measured data, as listed in Table 1. In this work, the renormalization factor Nr for the real parts of optical potentials is equal to 1.0 and the imaginary parts are the same for both the FDA and ADA approximations at each bombarding energy. As illustrated in Fig. 3, the angular distribution analyses from the optical model calculations are compared with the $^{12}C - ^{12}C$ elastic scattering data [3]. The results point out that the ADA real parts of the optical potentials describe the data better than that from the FDA calculations, especially for the large angles. One can note that the oscillations in the angular distributions at the forward and backward angles are the results of the incident wave functions scattering from the potential at the surface region and the central part, respectively. Consequently, the good description to data over the wide range of angles indicates that the strength and shape from the surface down to the center of the ADA potential is relevant to the $^{12}C + ^{12}C$ realistic interaction at low energies. As a result, the ADA regime is more reasonable to describe the nuclear medium or the nuclear environment in which two interacting nucleons are embedded at low bombarding energies than the FDA regime. Therefore, a conclusion has been drawn from the analysis is that the $^{12}C + ^{12}C$ reaction dynamic at low energies below 10 MeV per nucleon associates with the adiabtic process.

Table 1. The parameters of the imaginary part of the optical potential

Bombarding energy (MeV)	W_0 (MeV)	R_I (fm)	a_I (fm)
121.6	4.479	1.403	0.333
83.3	8.675	1.388	0.364
50.0	16.953	1.214	0.587

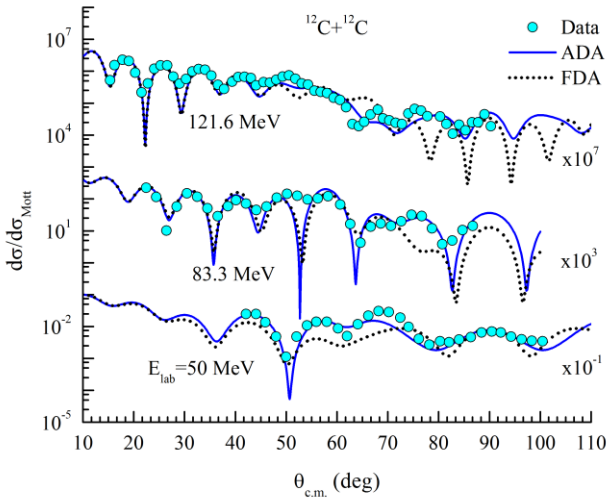

Figure 3. The elastic angular distributions for $^{12}C + ^{12}C$ system at low energies. The data are taken from Ref. [3]

CONCLUSION

The aim of this work is to investigate the nuclear medium during the colliding process of the $^{12}C + ^{12}C$ system at energy region below 10 MeV per nucleon. Both frozen and adiabatic density approximations are used for describing the nuclear medium in the folding procedure. The results obtained from the elastic scattering analysis of the $^{12}C + ^{12}C$ system with the optical model potential figure out that the adiabatic density approximation provides a better fit to data than the frozen density approximation. We conclude that the evolution of nuclear medium during the $^{12}C + ^{12}C$ approaching process at low energies is relevant to the adiabatic regime. In the further plan, the double folding potential within the adiabatic regime is applied to study the $^{12}C + ^{12}C$ fusion at Gamow window.

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Nghiên cứu mật độ hạt nhân dựa trên phân tích tán xạ đàn hồi $^{12}C + ^{12}C$ ở vùng năng lượng thấp

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TÓM TẮT

Mật độ hạt nhân được nghiên cứu thông qua phân tích tán xạ đàn hồi $^{12}C + ^{12}C$ ở vùng năng lượng dưới 10 MeV/nucleon dựa trên mẫu quang học hạt nhân. Trong đó, hai xấp xỉ được sử dụng để nghiên cứu yếu tố này là xấp xỉ "frozen" và "adiabatic". Thế hạt nhân trong phân tích này được xây dựng từ mẫu folding kép dựa trên các phiên bản mới nhất của tương tác phụ thuộc mật độ CDM3Y3 và hàm sóng trạng thái của các hạt nhân tương tác. Các kết quả thu được từ việc phân tích tán xạ đàn hồi $^{12}C + ^{12}C$ cho thấy xấp xỉ "adiabatic" mô tả tốt số liệu thực nghiệm hơn so với xấp xỉ "frozen".

Từ khóa: *tán xạ đàn hồi, mẫu quang học, xấp xỉ "adiabatic"*

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